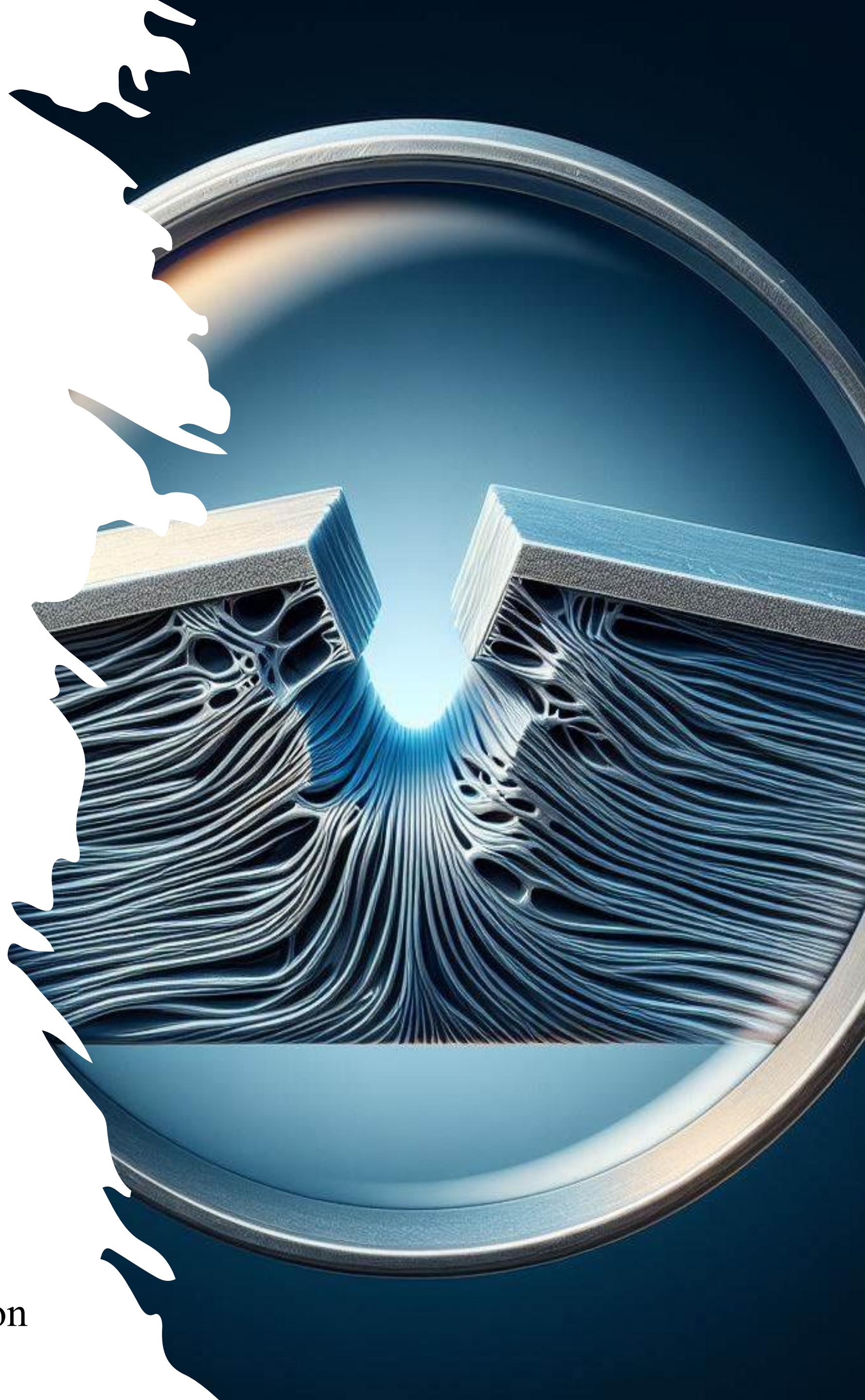
Stress Concentration

Stress Singularity

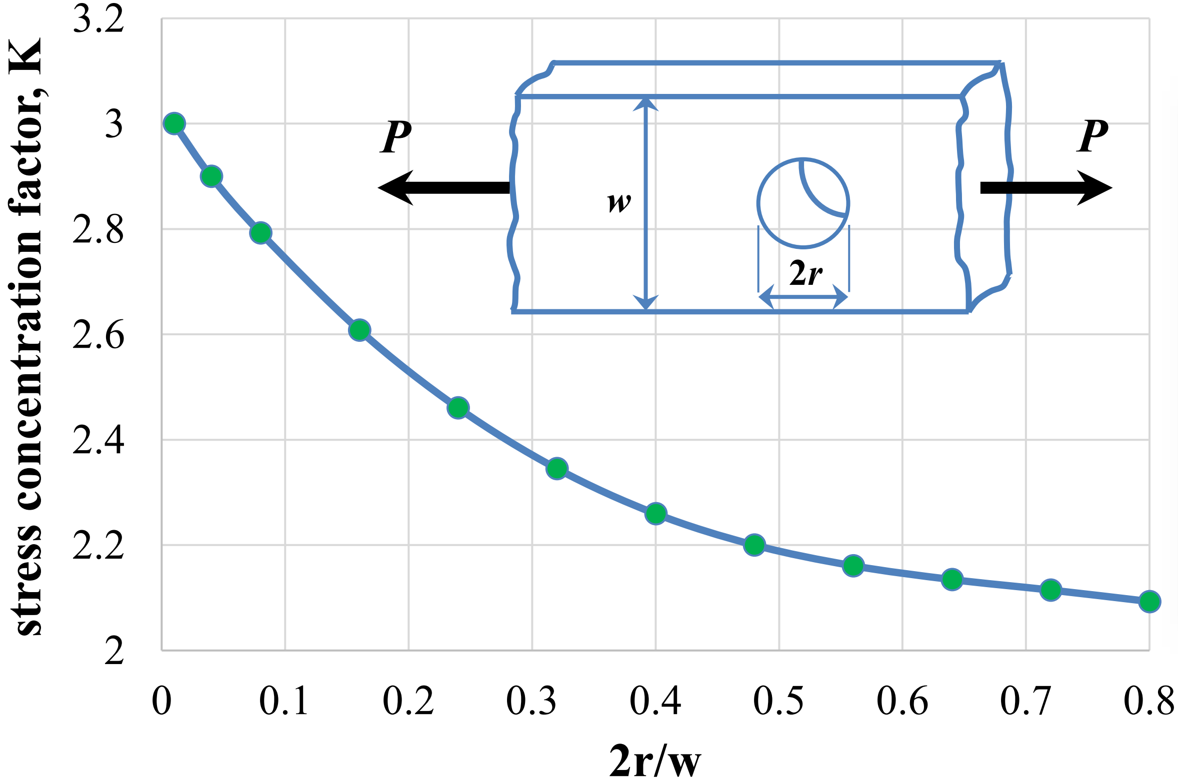


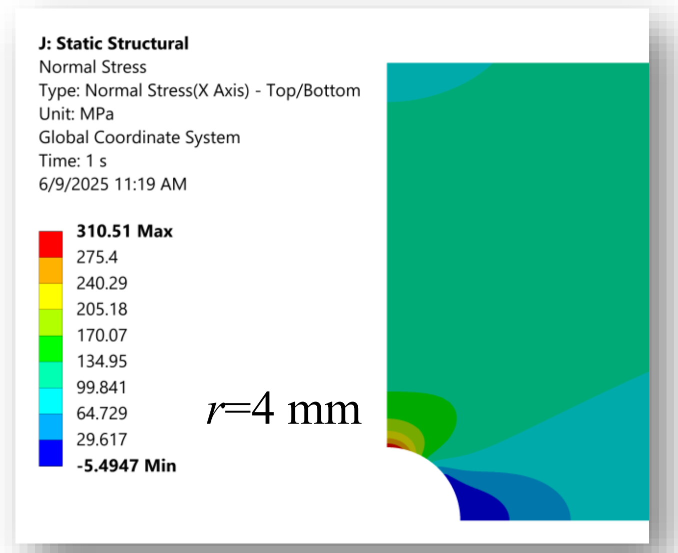
- This set of slides is a part of a larger and more comprehensive video on **Best Practices in Finite Element Analysis** coming soon on MechCADemy!
- So, stay tuned! Please, subscribe to MechCADemy on YouTube so you won't miss it!
- Watch the video "Theory of Finite Element Analysis, 8 simple and practical steps (watch before your next FEA)" on MechCADemy

Stress Concentration

- We cover this topic in courses such as Mechanics of Materials, or Design of Machine Elements.
- A very famous case is a plate with a hole in it.
- Specific values of K are generally reported in handbooks related to stress analysis or machine design books.

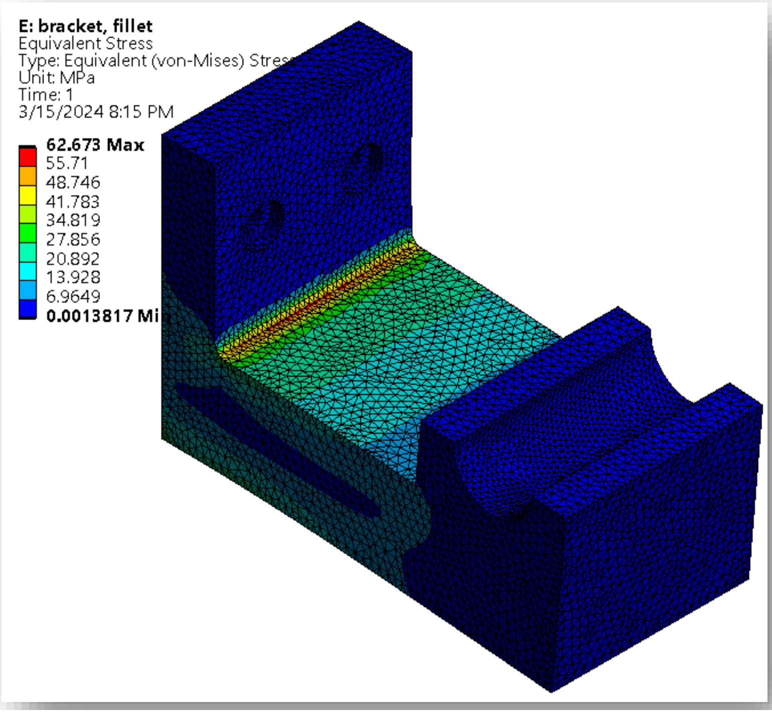
$$K = \frac{\sigma_{max}}{\sigma_0} \qquad \sigma_0 = \frac{P}{(w-2r)t}$$

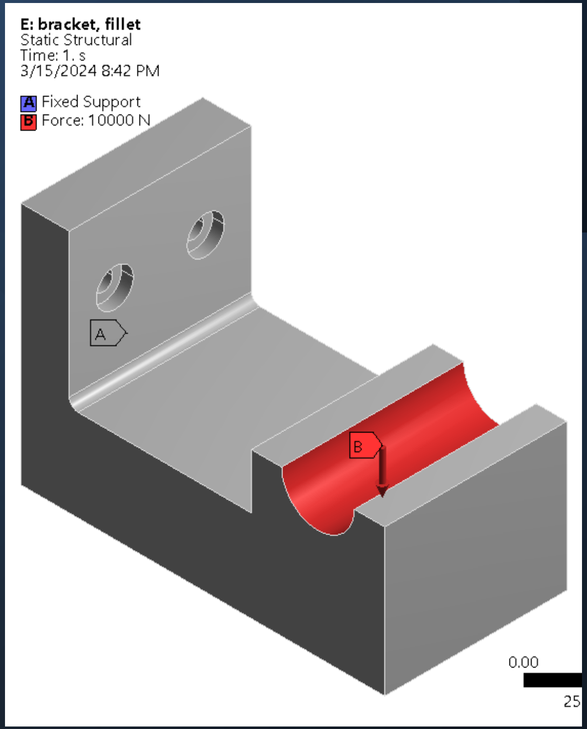


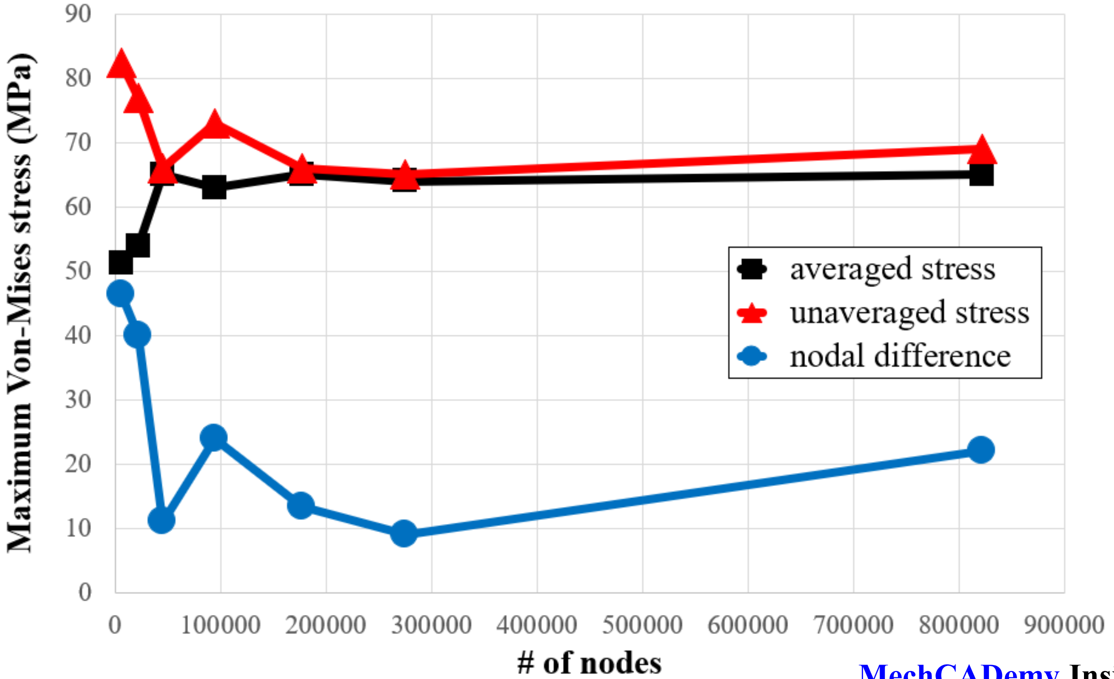


Stress Concentration

- stress converges in the presence of stress concentration.



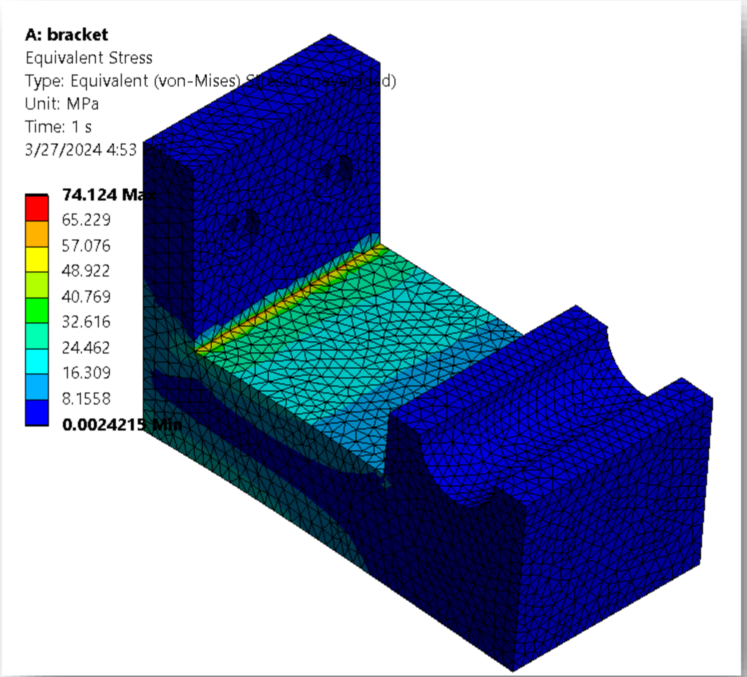


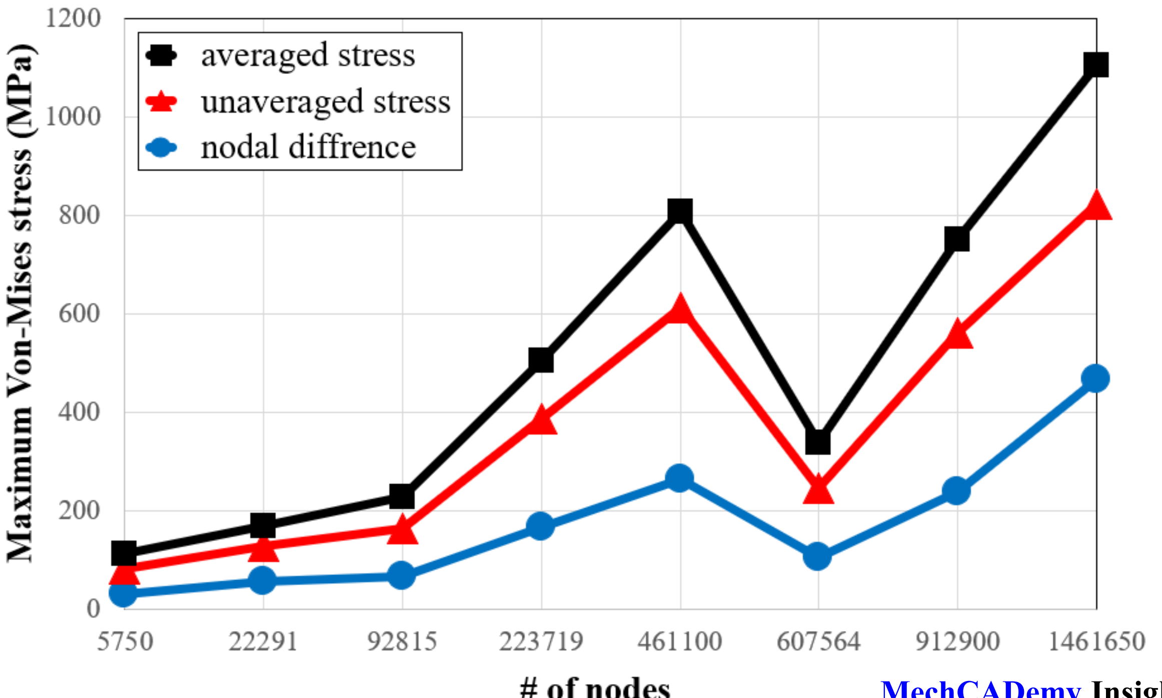


MechCADemy Insights

Stress Singularity

- There are times when the results don't converge no matter how refined the mesh is!!

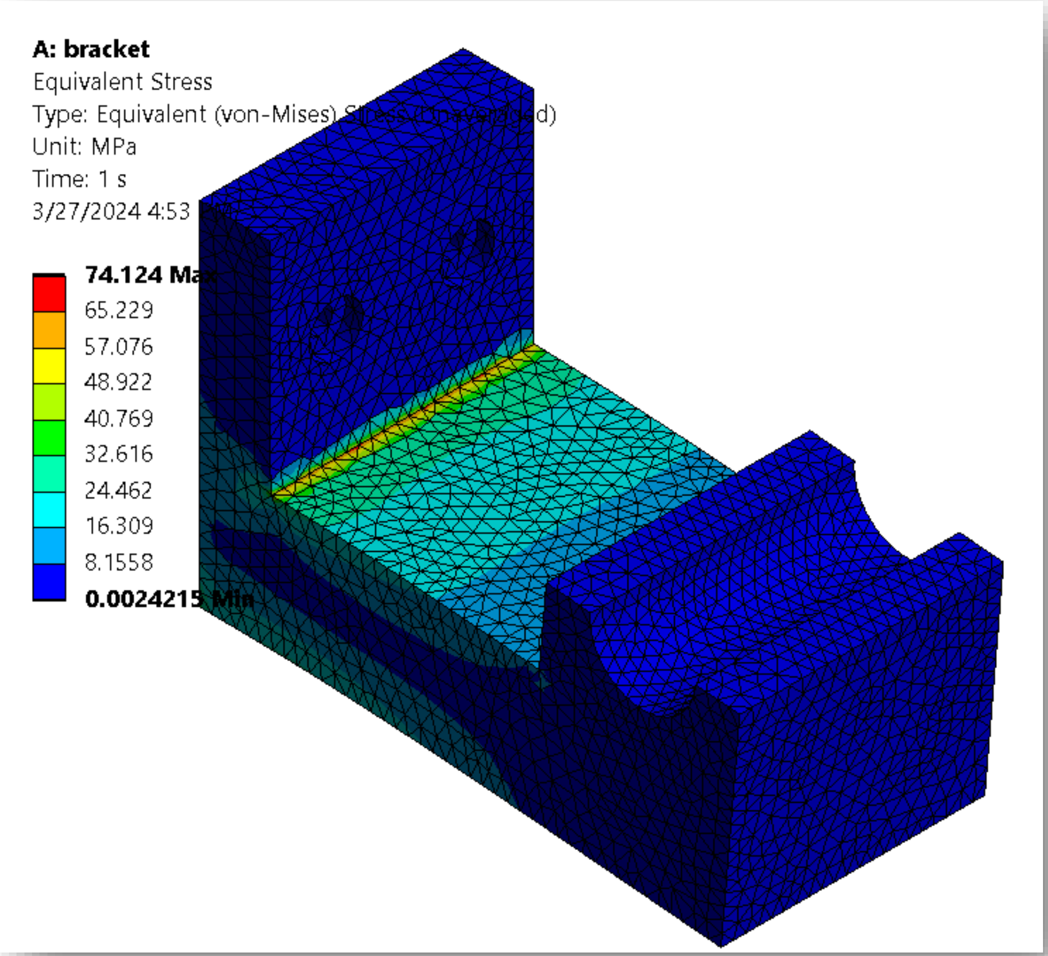




MechCADemy Insights

Sources of stress singularity

- Simplified CAD models (fillet requires more refined mesh!, so we remove it if justified)
- An example is Sharp re-entrant corners or inside corners.



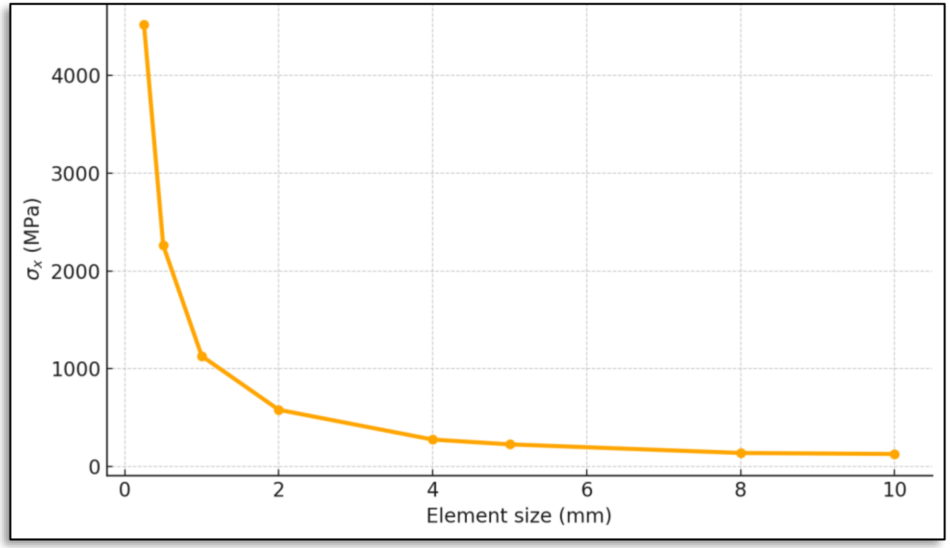
Sources of stress singularity

❑ Point loads

- Stress singularity is not the result of FEA!
- It is inherited in the physics of the problem.
- In 2D models, for point loads that are normal to an edge, using theory of elasticity, we can prove that:

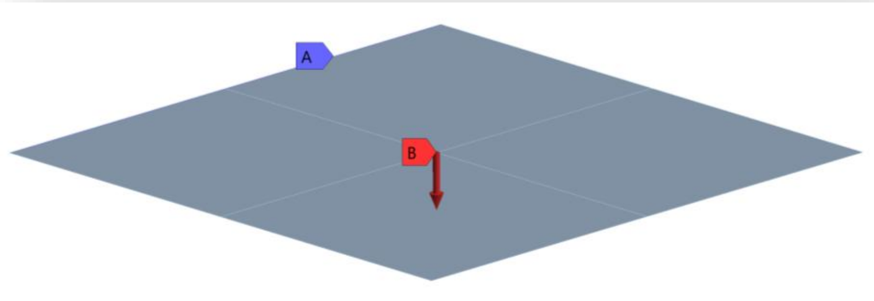
$$\sigma \propto \frac{1}{r}$$

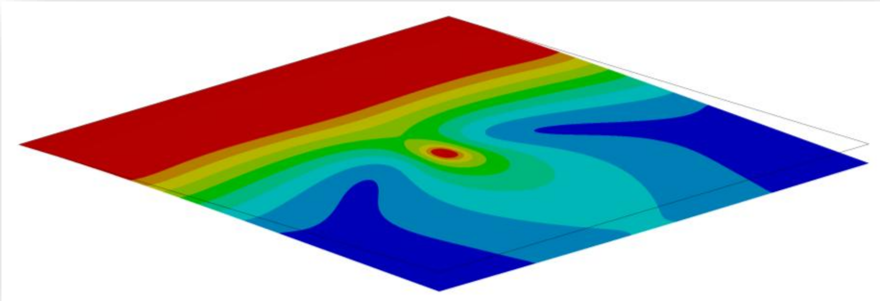
- This is based on *Flamant Solution* for a concentrated force on a half-plane [J.R. Barber, Elasticity, Ed. 3, pp. 173-174].



Sources of stress singularity

- But point loads don't always result in stress singularity
- Point load on a beam element or normal to a shell element doesn't create singularity!





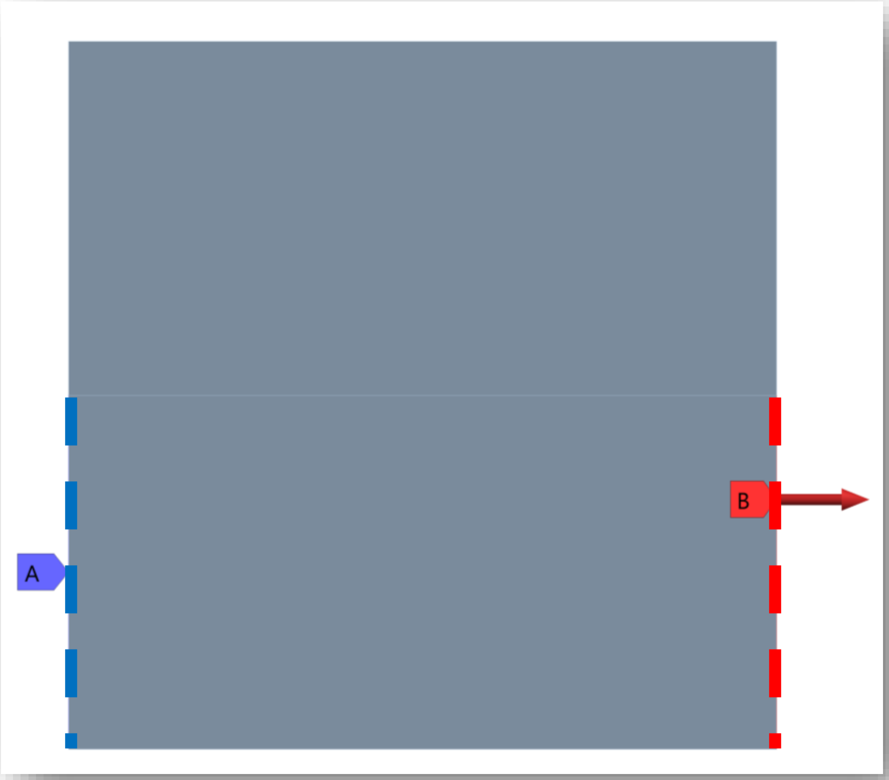
- Beam and shell theories pre-average the same load over the cross-section or thickness, keeping stresses finite and mesh-independent; the singularity is a consequence of the modelling level, not the load itself.

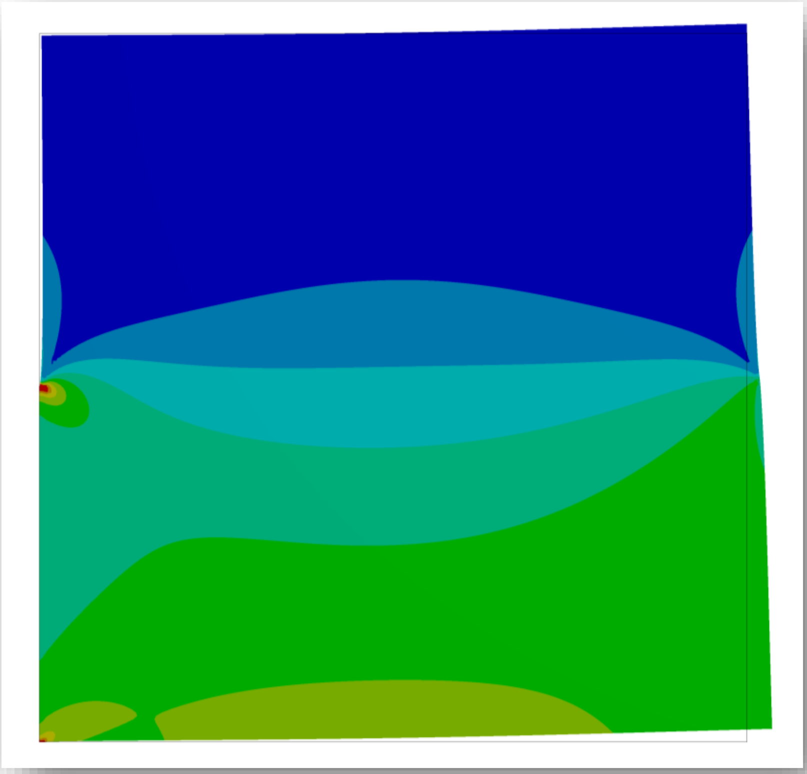
$$\sigma_{xx} = My/I;$$
$$\tau_{xz} = VQ/Ib$$

- However, a point load applied in the plane of a shell *will* cause a singularity.
- A point load in the plane of a shell (3D surface), acts on the membrane feature of the shell, and stress is being calculated using theory of elasticity, but changing with $1/r$ instead of $1/r^2$.

Sources of stress singularity

- ❑ Sudden change in the boundary condition





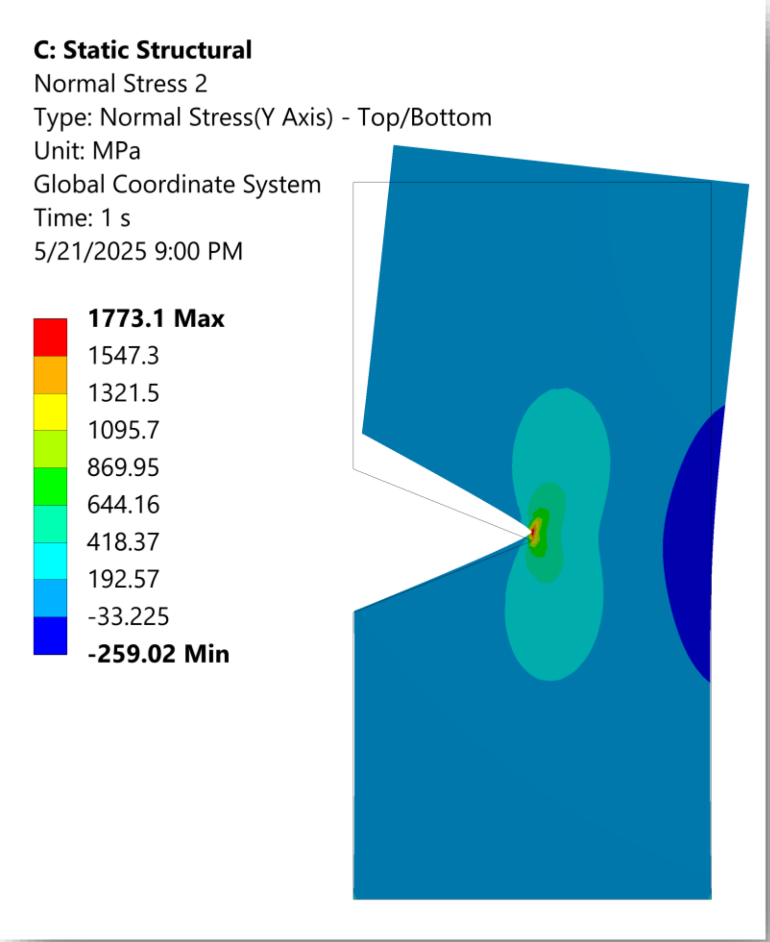
Sources of stress singularity

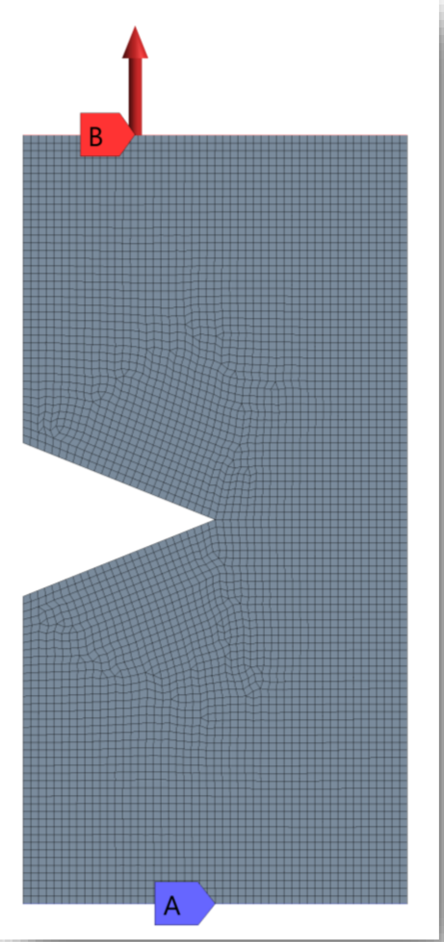
❑ Cracks!

- It can be shown [J.R. Barber, Elasticity, Ed. 3, pp. 173-174]:

$$\sigma \propto \frac{1}{\sqrt{r}}$$

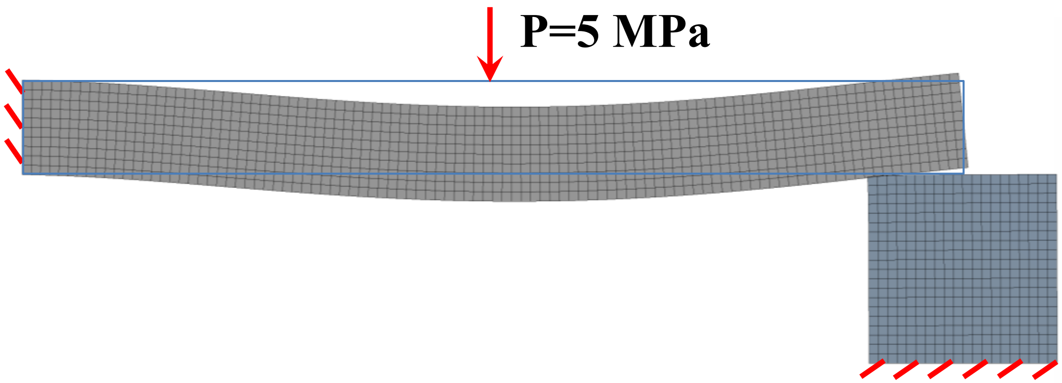
Where r is measured from the crack tip.

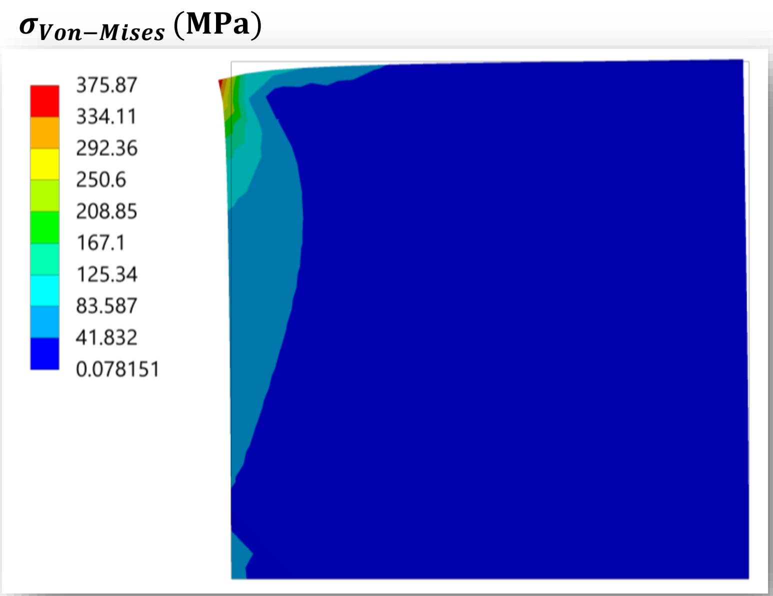


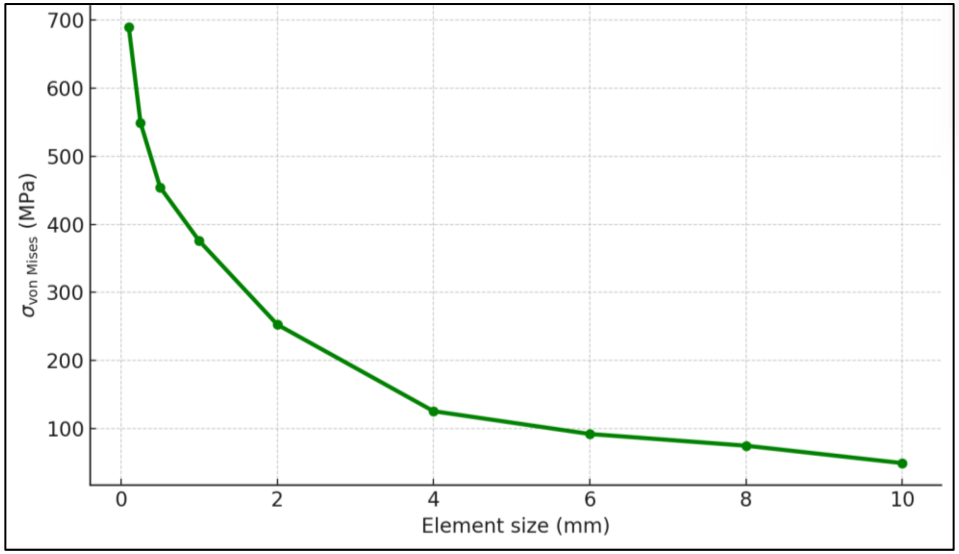


Sources of stress singularity

- Singularity at contact







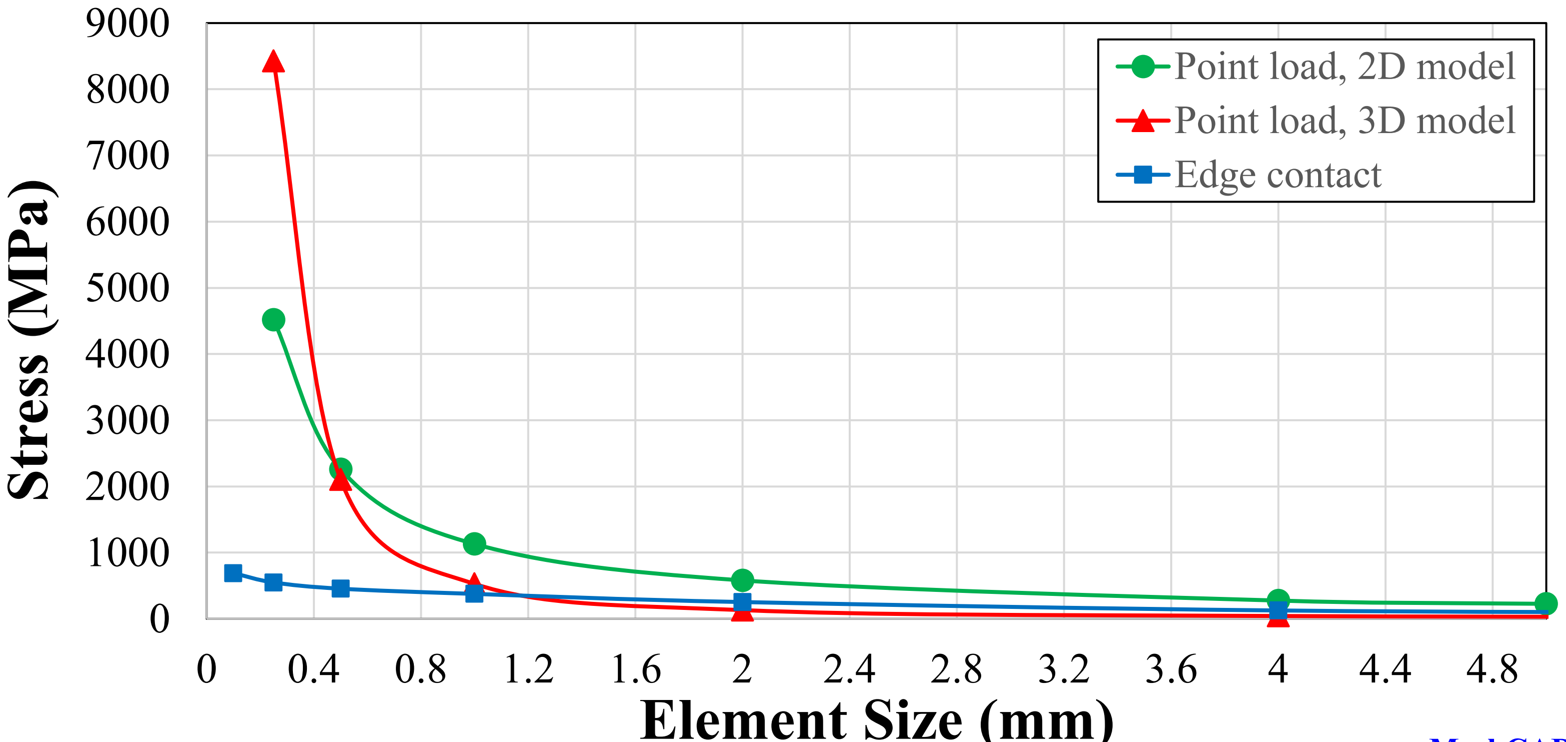
- It can be shown:

$$\sigma \propto \frac{1}{\sqrt{r}}$$

Where r is measured from the edge.

Not all singularities are similar!

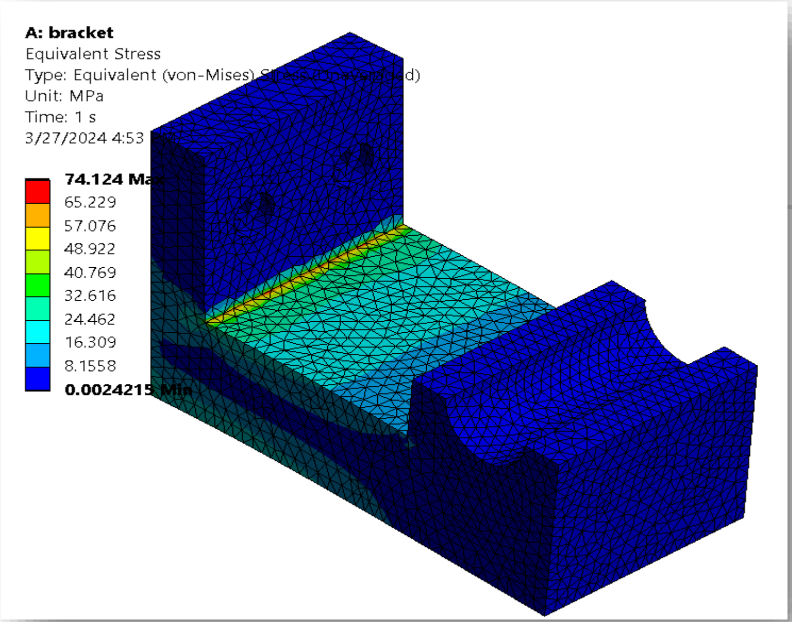
- Singularities have different orders!
- Singularity of point load normal to a surface in a 3D model is of order $1/r^2$
- Singularity of point load normal to a surface in a 2D model is of order $1/r$
- Singularity of edge contact is of order $1/\sqrt{r}$
- Remember, these are related to the theory of elasticity, not FEA!

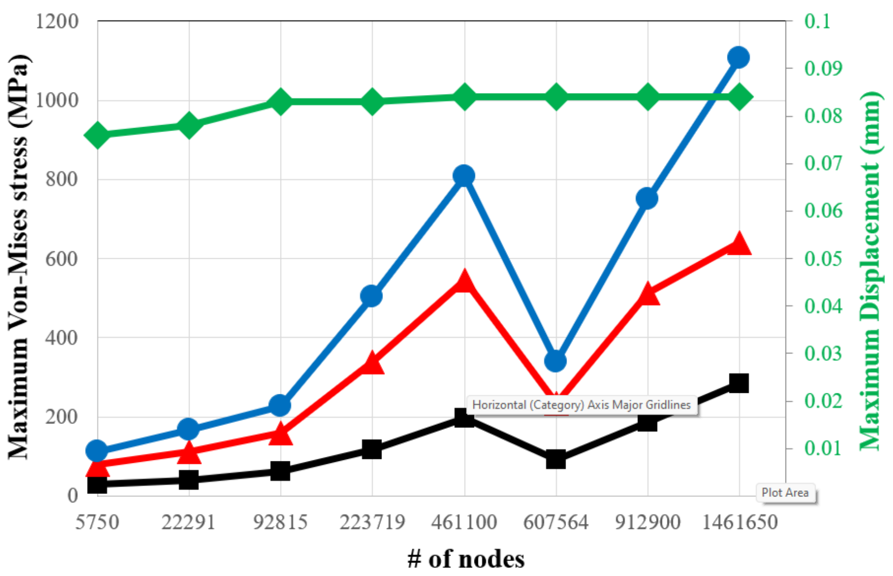


MechCADemy Insights

VERY IMPORTANT MESSAGE

Displacement/Temperature are not singular.





MechCADemy Insights

- **Can force (reaction force) be singular?**
- **Can strain be singular?**

Please respond in the comments!

A brain teaser

for more enthusiastic audience!

- Stress singularity of a point load on a surface of a 3D solid is of order $1/r^2$
- Kelvin–Boussinesq solution for a point load on an infinite surface of a 3D solid predicts 1/r singularity for ***displacement***! But why isn't displacement singular in FEA!?

$$\boxed{\sigma_{ij}(\mathbf{r}) = -\frac{F}{8\pi(1-\nu)}\,\frac{n_i\delta_{jz}+n_j\delta_{iz}+(1-2\nu)\,n_in_jn_z}{r^2}} \qquad r=|\mathbf{r}|,\ n_i=\frac{r_i}{r}$$

$$\boxed{u_i(\mathbf{r}) = \frac{F}{16\pi\mu(1-\nu)\,r}\Big[(3-4\nu)\,\delta_{iz}+n_in_z\Big], \qquad r=|\mathbf{r}|,\ n_i=\frac{r_i}{r}}$$

Please respond in the comments!

Radial decay/growth of stress and displacement

Situation	$\sigma \sim r^m$	$u \sim r^{m+1}$
3-D point load	$m=-2$	$u \sim r^{-1}$ (singular)
Crack tip (Modes I–III)	$m=-\frac{1}{2}$	$u \sim r^{+\frac{1}{2}}$ (finite)
Edge or line contact	$m=-\frac{1}{2}$	$u \sim r^{+\frac{1}{2}}$ (finite)

- Do we have a singularity for the displacement for point load on a 2D half plane? If we do, what is the order?!

Best Practices in Finite Element Analysis

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